Persistent currents in high dimensions

1997 J. Phys.: Condens. Matter 9 8257

(http://iopscience.iop.org/0953-8984/9/39/011)

J. Phys.: Condens. Matter **9** (1997) 8257–8274. Printed in the UK PII: S0953-8984(97)80102-6

Persistent currents in high dimensions

Jörn Barth† and Alexander Altland†‡

† Institut für Theoretische Physik, Universität zu Köln, Zülpicher Strasse 77, 50937 Köln, Germany

‡ Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, UK

Received 5 December 1996, in final form 16 May 1997

Abstract. We introduce a model for the persistent current carried by spinless fermions moving in a ring with a high-dimensional cross section. The effects of both disorder and electron–electron interaction are considered. It is found that the non-interacting system behaves like previously considered low-dimensional models. The more complicated interacting/disordered case is analysed by means of a functional integral which is evaluated in the limit of infinite dimensionality by expanding in the number of transverse channels. To leading order in this expansion scheme, the Coulomb interaction does not affect the persistent current. It is found that the insensitivity to interaction effects is due to the absence of local contributions to the Coulomb vertex in our model (which in turn is a consequence of the neglect of the electron spin). It is argued that the physical mechanism suppressing the interaction in the high-dimensional spinless model applies to the analogous low-dimensional case as well.

1. Introduction

Some years ago it was shown experimentally [1] that metallic rings subject to a magnetic field may carry a ground-state equilibrium current. Whereas experiments probing the existence of these so-called persistent currents necessitate the fabrication of mesoscopic sub-micron devices and have not been feasible until recently, the theoretical analysis of the phenomenon has a long history that dates back to the thirties [2]. In spite of substantial theoretical efforts, however, there is still a mismatch of more than one order of magnitude between the theoretically predicted and the experimentally observed value of the current. It is now more or less generally believed that the key to the solution of the problem must lie in taking proper account of the Coulomb interaction between the electrons. Whereas early attempts to explain the magnitude of the experimentally observed current [3–6] have focused on the effects of disorder, more recent theoretical analyses emphasize the role of interactions. Roughly speaking, the group of theoretical approaches can be divided into (i) perturbative diagrammatic analyses [7, 8], (ii) exact-diagonalization procedures [9, 10], (iii) Hartree–Fock-type calculations [11, 12] and (iv) the analysis of strictly one-dimensional (1d) systems [13–20]. Yet in spite of the variety of different approaches, the present state of the theory still does not allow one to safely draw conclusions as to the role of the Coulomb interaction. For one thing, none of the above-mentioned approaches can be applied to a *quantitative* analysis of the experimental data: due to a complex disorder–renormalization behaviour of the effective Coulomb interaction, the direct diagrammatic approach to problem (i) has not yet led to conclusive results [21]. The model systems underlying the numerical analyses (ii) are generally too small to account for the presence of long-range diffusion modes which govern the transport in the experimental systems. The

0953-8984/97/398257+18$19.50

self-consistent Hartree–Fock approach (iii) excludes those diagrams which cause the most severe divergencies in the general diagrammatic analysis (i); their role remains unclear. Finally, it is impossible to describe the fundamental transport mechanism in mesoscopic *multi-channel* systems: diffusion, within strictly 1d models (iv). Moreover, 1d systems do not possess an extended Fermi surface, which implies that the Coulomb interaction plays a role fundamentally different from that in the higher-dimensional cases. Hence, it is impossible to reach conclusions based on the physics of 1d systems as to the experimental situation.

Even if one is primarily interested not in a quantitative explanation of the experimental data but in obtaining an answer to the more fundamental question of the influence of Coulomb interactions on the thermodynamics of mesoscopic systems, the present state of the theory is incomplete. The point is that the above-mentioned theoretical approaches yield differing results, this being an indication of the fact that they do not describe a common physical scenario. Even within the restricted class of strictly 1d models, the situation is diverse. Depending on the microscopic definition of the model considered, the spectrum of predictions ranges from a drastic interaction-induced enhancement of the current [13] (up to its value in the clean limit), over a moderate increase [18–20], to a suppression [18–20]. In the more recent stages of the debate, evidence for the importance of the *electron spin* has accumulated [11]. Depending on the inclusion or exclusion of the electron spin, calculations performed within one and the same model gave different signs for the interaction-induced contribution to the current. Altogether, these findings indicate that (a) the theoretical analysis of the current in the (experimentally realized) multi-channel/disordered/interacting case is notoriously difficult, and (b) the current in 1d seems to have non-universal features in the sense that its qualitative behaviour is susceptible to changes in a variety of system parameters.

To understand better the role of the Coulomb interaction in the multi-channel case, it may be helpful to look for simplified model systems which still inherit the essential features of the full problem. In the present paper we discuss a model of interacting electrons in multi-channel Aharonov–Bohm rings which meets this criterion. The idea is to consider rings with a cross section of high dimensionality $d - 1 \gg 1$ (rather than $d = 2$ in reality). We are thus modelling the opposite extreme to the 1d case, namely systems with a quasi-infinite number of channels. Models of high dimensionality have been introduced earlier [22, 23], and applied to the analysis of disordered interacting electron systems [24, 25]. Their key advantage in the present context is that (i) in the absence of interactions the current behaves like it does in the real multi-channel systems (in particular, there is long-range diffusive motion along the perimeter of the ring), which means that the model shares at least some of its essential properties with real systems, and (ii) the existence of a small expansion parameter, namely d^{-1}, which leads to a substantial simplification of the analysis of interactions.

In the present paper we discuss the model in its simplest form, i.e. we consider the case of spinless fermions in the limit $d \to \infty$ (i.e. we do not account for d^{-1}-corrections). As a result we find that the role of spin *is* of central importance. More precisely, we find that in the spinless case interactions do not affect the current at all. It will become obvious from the analysis below that the inclusion of spin would give rise to qualitatively different interaction contributions which would influence the current. To complete the analysis, one should thus include the electron spin. Moreover the robustness of the predictions obtained in the large-d limit should be checked by calculating d^{-1}-corrections.

Nevertheless we think that it is worthwhile to present the model now, in its present simple form. For one thing, we believe that our results can help in providing a better

understanding of the role of spin in the multi-channel case. Second, we hope that the field theoretical formalism developed in this paper will prove to be of interest in its own right, insofar as it may serve as a formal starting point for the analysis of related problems in high-dimensional interacting disordered Fermi systems.

The paper is organized as follows. In section 2 we introduce the model, and discuss its physics in the clean non-interacting limit. In section 3 we introduce our field theoretical machinery and consider the effect of disorder while interactions are still ignored. Section 4 deals with the influence of the Coulomb interaction. In section 5 we interpret our result in a qualitative manner, and discuss its generalization to low-dimensional cases. We give our conclusions in section 6.

2. Definition of the model and the clean non-interacting limit

In this section we define the model, calculate the current in the simple case where both interaction and disorder are switched off, and compare the result with previous analyses of persistent currents in clean multi-channel geometries.

We consider a model of spinless fermions defined on a d-dimensional lattice of constant spacing a. The system is highly anisotropic in the sense that its extension in the dth dimension is $L_\parallel$, and $L_\perp \ll L_\parallel$ in the remaining $d-1$ dimensions. We impose periodic (hard-wall) boundary conditions in the longitudinal, $\parallel$ (transverse, $\perp$) direction, thereby giving the system the geometry of a torus of circumference $L_\parallel$ and cross section $S = L_\perp^{d-1}$ (cf. figure 1). A persistent current is induced by applying an Aharonov–Bohm flux piercing the ring.

The Hamiltonian representing the system reads

$$\hat{H} = \hat{H}_K + \hat{H}_V + \hat{H}_I \tag{1}$$

where

$$\begin{aligned}
\hat{H}_K &= \sum_{\langle x,y\rangle} \boldsymbol{a}^\dagger(x) H_K(\phi; x, y) \boldsymbol{a}(y) \\
H_K(\phi; x, y) &= t \exp\left(\mathrm{i} \frac{\phi}{\phi_0} \frac{2\pi}{L_\parallel} (x_\parallel - y_\parallel) \right) \times \begin{cases} 1 & x_\perp = y_\perp \\ d^{-1/2} & x_\perp \neq y_\perp \end{cases} \\
\hat{H}_V &= \sum_x \boldsymbol{a}^\dagger(x)(V(x) - \mu)\boldsymbol{a}(x) \\
\hat{H}_I &= \frac{U}{2d} \sum_{\langle x,y\rangle} \boldsymbol{n}(x)\boldsymbol{n}(y)
\end{aligned} \tag{2}$$

where x, y are lattice vectors, $\boldsymbol{a}$ denotes a fermionic annihilation operator, $\boldsymbol{n} = \boldsymbol{a}^\dagger \boldsymbol{a}$, $\sum_{\langle x,y\rangle}$ is a sum over nearest neighbours, ϕ the magnetic flux piercing the ring, ϕ_0 the elementary flux quantum, μ the chemical potential (which we set to a value close to zero corresponding to an approximately half-filled band), and t and U are positive constants governing the strength of the hopping matrix element and the Coulomb interaction, respectively. The scaling factor $d^{-1/2}$ (d^{-1}) multiplying the transverse hopping matrix element (the interaction Hamiltonian) is needed to keep the expectation value of the kinetic (interaction) energy finite in the limit $d \to \infty$ [22]. In order to allow for non-trivial diffusion in the tangential direction, the $\parallel$-hopping matrix elements are not rescaled. This anisotropic treatment of the ring is needed to obtain a non-vanishing persistent current in the $d \to \infty$ limit. Disorder is

introduced via a Gaussian potential V specified by the correlator

$$\begin{aligned} \overline{V(x)} &= 0 \\ \overline{V(x)V(y)} &= \frac{1}{2\pi\nu\tau}\delta(x-y) \end{aligned} \tag{3}$$

where $\overline{(\cdots)}$ denotes the disorder average, $\delta(x-y) = a^{-d}\delta_{x,y}$ is the lattice version of the δ-distribution, τ the mean elastic scattering time, and ν is the density of states per volume.

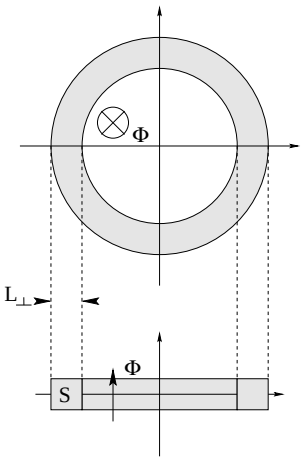

Figure 1. The high-dimensional model system. The circumference of the ring is given by $L_\parallel$.

In order to get some feeling for the basic physics of the model, let us consider the simple case where both interaction and disorder are switched off, $V = U = 0$, and the temperature is set to 0. Under these conditions, $\hat{H}$ takes the form of a clean tight-binding Hamiltonian which can readily be diagonalized. Its eigenvalues read

$$\epsilon(k) = -2t\left(\sum_{i=1}^{d-1}\frac{1}{\sqrt{2d}}\cos(k_i a) + \cos\left(\left[k_d - \frac{\phi}{\phi_0}\frac{2\pi}{L_\parallel}\right]a\right)\right) = \epsilon_\perp(k_\perp) + \epsilon_\parallel(k_\parallel) \tag{4}$$

where $\epsilon_\perp$ ($\epsilon_\parallel$) denotes the first (second) term on the rhs of the first equation,

$$\begin{aligned} k_\parallel &= k_d - \frac{\phi}{\phi_0}\frac{2\pi}{L_\parallel} \\ k_d &= \frac{2\pi n_d}{L_\parallel} \qquad n_d \in \mathcal{Z} \\ k_i &= \frac{\pi n_i}{L_\perp} \qquad i = 1\ldots d-1 \qquad n_i \in \mathcal{N} \end{aligned} \tag{5}$$

and $k_\perp^{\mathrm{T}} = (k_1, \ldots, k_{d-1})^{\mathrm{T}}$. The persistent current of clean two-dimensional tight-binding models has been discussed previously in reference [26]. To demonstrate that the number of transverse dimensions does not affect the basic physics, let us briefly outline an analogous calculation for the high-dimensional case.

The persistent current $I(\phi)$ of a clean multi-channel ring is a quantity which depends sensitively on various system parameters such as the chemical potential and the system's geometry. It readily averages to zero upon slight variation of any of these parameters. A

more robust quantity is the *typical* value of the harmonics of the current, $(\overline{I_l^2})^{1/2}$, where the harmonics are defined by the expansion [27]

$$I(\phi) = \sum_l I_l \exp\left(\mathrm{i}\frac{\phi}{\phi_0} 2\pi l\right)$$

and $\overline{(\cdots)}$ denotes an average over a (microscopically small) range of system sizes and/or values of the chemical potential. In complete analogy to reference [26] we obtain

$$\overline{I_l^2} = \overline{I_{d=1,l}^2} \sum_{k_\perp} \sin^2(k_f(k_\perp)a) = \overline{I_{d=1,l}^2} \sum_{k_\perp} \left(1 - \left(\frac{\epsilon_\perp(k_\perp)}{2t}\right)^2\right) \tag{6}$$

where $\overline{I_{d=1,l}^2}$ denotes the square of the typical current of a strictly 1d ring [26], and

$$k_f(k_\perp) = a^{-1}\cos^{-1}[(\mu - \epsilon_\perp(k_\perp))/(2t)]$$

is the 'Fermi momentum' associated with the 'longitudinal' part of the kinetic energy $\mu - \epsilon_\perp(k_\perp)$. (Equation (6) relates to the case where $\mu = 0$.)

Expressions like (6)—that is, sums of the type

$$\sum_{k_\perp} f(\epsilon_\perp(k_\perp))$$

where f is some smooth function—will be encountered repeatedly in the following. In general, the complex structure of the multi-dimensional Fermi surface renders the computation of these momentum sums difficult. In the case of large d, however, this source of complexity turns into an advantage in the sense that *statistical* arguments can be applied to convert the sum into an integral over a smooth distribution function [23]. As is reviewed in the appendix,

$$\sum_{k_\perp} f(\epsilon_\perp(k_\perp)) \stackrel{d\to\infty}{\simeq} \frac{N_\perp}{\sqrt{2\pi}t} \int_{-\infty}^{\infty} \mathrm{d}\epsilon \ \exp\left(-\frac{\epsilon^2}{2t^2}\right) f(\epsilon) \tag{7}$$

where $N_\perp = (L_\perp/a)^{d-1}$ denotes the number of transverse modes. Applying this identity to the computation of (6), we obtain

$$\overline{I_l^2} \simeq N_\perp \overline{I_{d=1,l}^2}.$$

This result conforms with the findings of reference [26] in the sense that the typical multi-dimensional current is obtained from the one-dimensional current by a multiplication by the square root of the number of transverse channels. Apart from the fact that this number grows exponentially with d, the large-d limit does not influence the behaviour of the current.

3. Disorder

In the preceding section we have seen that the dimensionality of the clean model does not affect its basic physics. How does the situation change if disorder is added? Towards the end of this section it will become clear that the presence of disorder does not alter the conclusion drawn above: the persistent current for large d behaves similarly to its low-dimensional analogue. However, the calculations performed in the following will be technically quite involved, and the same applies to the next section where the role of the Coulomb interaction will be considered. Readers who are not willing to make their way through all of these technicalities are invited to proceed directly to section 5, where our results are discussed in a qualitative and intuitively accessible way.

Persistent currents in disordered metallic rings have been the subject of numerous theoretical analyses [3–6]. In contradiction to the experimental findings, the first calculations of the persistent current in diffusive rings gave a vanishingly small result. In subsequent work it turned out [3–5] that a non-negligible value can only be obtained if the strict conservation of the number of particles in each individual ring is carefully taken into account. A formula implementing this condition into an effectively grand-canonical description reads [3–5]

$$I = -\frac{\Delta}{2}\,\partial_\phi \overline{\delta N^2} \tag{8}$$

where Δ is the mean level spacing at the chemical potential. Equation (8) relates the current to the fluctuations in the number N of (single-) particle states below the chemical potential (here and henceforth we set $\delta X := X - \overline{X}$). The derivation of this identity is based on a few general thermodynamic identities, and does not depend on specific system characteristics such as the dimensionality. We may thus apply (8) to a calculation of the persistent current in the high-dimensional case, and compare the result with previous work.

In the case under consideration, i.e. a weakly disordered multi-channel ring, it would be most efficient to calculate the current by means of diagrammatic perturbation theory. Here however we will take an alternative field theoretical route, thereby introducing the formalism to be applied below to the analysis of the more complex interacting/disordered problem. For the sake of future reference we perform the calculation of $\overline{\delta N^2}$ within the finite-temperature formalism (although the temperature will be set to zero throughout this section).

The information needed to calculate the current is contained in the grand-canonical potential Ω and fluctuations thereof. To compute Ω we employ the replica trick and functional integration over anticommuting variables. As is shown e.g. in [29], Ω can be represented as

$$\begin{aligned}
\Omega &= \lim_{N\to 0} N^{-1} \int \mathcal{D}\psi \;\; \exp(-S_1[\psi] - S_2[\psi]) \\
S_1[\psi] &= -\beta\psi^\dagger(\mathrm{i}\hat{\omega} + \mu - H_K(\phi) - V)\psi \\
S_2[\psi] &= \frac{\beta U}{2d} \sum_{\langle x,y\rangle} \sum_{n_i} \sum_{\alpha} \psi^{\alpha\dagger}_{n_1}(x)\psi^{\alpha}_{n_2}(x)\psi^{\alpha\dagger}_{n_3}(y)\psi^{\alpha}_{n_4}(y)\delta_{n1+n3,n2+n4}
\end{aligned} \tag{9}$$

where $\beta = (kT)^{-1}$, $\psi = \{\psi^\alpha_n(x)\}, \alpha = 1,\ldots,N, n \in \mathcal{Z}$, is a field of anticommuting variables, $\hat{\omega} = \{\omega_n\}$, $\omega_n = \pi(2n+1)T$, and a summation over all indices which are not indicated explicitly is understood. By the same methods it can be shown that the particle number fluctuations $\overline{\delta N^2} = \overline{(\partial_\mu \delta\Omega)^2}$ entering the expression for the (non-interacting) persistent current can be computed as

$$\begin{aligned}
\overline{\delta N^2} &= \beta^{-2} \lim_{N\to 0} \partial^2_{\mu_1,\mu_2}\Big|_{\hat{\mu}=\mu\cdot 1_N} \overline{Z[\hat{\mu}]} - \overline{N}^2 \\
Z[\hat{\mu}] &= \int \mathcal{D}(\psi,\psi^\dagger)\; \overline{\exp(-S_1[\psi,\hat{\mu}])} \\
S_1[\psi,\hat{\mu}] &= -\beta\psi^\dagger(\mathrm{i}\hat{\omega} + \hat{\mu} - H_K(\phi) - V)\psi
\end{aligned} \tag{10}$$

where $\hat{\mu} = \mathrm{diag}(\mu_1,\ldots,\mu_N)$ and 1_N is the unit matrix in replica space. To compute (10) we employ the formalism of the non-linear σ-model. For comprehensive introductions into the replicated finite-temperature version of this model, we refer the reader to the literature (e.g. [30, 32] and references therein). Here we restrict ourselves to a concise description of its construction where the emphasis will be on the role of the dimensionality.

As a first step we introduce a new field Ψ as follows:

$$\psi \to \Psi = \begin{pmatrix} \psi \\ \psi^* \end{pmatrix}. \tag{11}$$

Expressed in terms of Ψ, the action reads

$$S_1[\Psi, \hat{\mu}] = -\frac{\beta}{2}\Psi^\dagger \left[\mathrm{i}\hat{\omega} + \hat{\mu} - \begin{pmatrix} H_K(\phi) & 0 \\ 0 & H_K(-\phi) \end{pmatrix} - V \right] \Psi$$

where the matrix structure corresponds to the two-component space appearing in (11). The latter will henceforth be called the T-space. The motivation for the component doubling in (11) is that the newly introduced Ψ obeys the symmetry relation

$$\Psi^* = M\Psi \qquad M = \mathrm{i}\sigma_2^{\mathrm{T}}. \tag{12}$$

This identity will enable us to take account of the time-reversal symmetry of the model (by σ_x^{T}, $x = 1, 2, 3$, we mean the Pauli matrices acting in T-space) [33].

We next average the functional over the disorder, thereby generating a contribution to the action which is quartic in Ψ. To decouple the latter we introduce an auxiliary field $Q = \{Q_{nm}^{\alpha\beta,ij}(x)\}$, where $\alpha, \beta = 1, \ldots, N$ ($n, m \in \mathcal{Z}$, and $i, j = 1, 2$) are the replica (Matsubara, T) indices. The symmetry of the Q-field is chosen so as to be compatible with (12): $Q^* = MQM^{-1}$. A Hubbard–Stratonovich transformation then leads to

$$\begin{aligned} \overline{Z[\hat{\mu}]} &= \int \mathcal{D}Q \ \mathcal{D}\Psi \ \exp(-S_1[Q, \Psi, \hat{\mu}]) \\ S_1[Q, \Psi, \hat{\mu}] &= -\frac{\beta}{2}\Psi^\dagger \left[G_0^{-1} + \frac{\mathrm{i}}{2\tau} Q \right] \Psi + \frac{\pi\nu}{8\tau} \operatorname{tr} Q^2 \\ G_0^{-1} &= \mathrm{i}\hat{\omega} + \hat{\mu} - \begin{pmatrix} H_K(\phi) & 0 \\ 0 & H_K(-\phi) \end{pmatrix}. \end{aligned} \tag{13}$$

The Gaussian integration over Ψ can now be performed, and we arrive at

$$\begin{aligned} \overline{Z[\hat{\mu}]} &= \int \mathcal{D}Q \ \exp(-S_1[Q, \hat{\mu}]) \\ S_1[Q, \hat{\mu}] &= -\frac{1}{2} \operatorname{tr} \ln \left[G_0^{-1} + \frac{\mathrm{i}}{2\tau} Q \right] + \frac{\pi\nu}{8\tau} \operatorname{tr} Q^2 \end{aligned} \tag{14}$$

where the trace extends over all indices (including the spatial ones). To compute (14) we employ a saddle-point approximation, i.e. we expand the action around the solutions of

$$\frac{\delta S_1[Q, \hat{\mu}]}{\delta Q(x)} = 0 \Leftrightarrow Q(x) = \frac{\mathrm{i}}{\pi\nu} \left[G_0^{-1} + \frac{\mathrm{i}}{2\tau} Q \right]^{-1} (x, x). \tag{15}$$

Equation (15) is solved by the manifold of matrices (cf. e.g. reference [30])

$$\begin{aligned} Q &= T^{-1}\Lambda T \qquad T \in \boldsymbol{G}/\boldsymbol{H} \\ \Lambda &= \{\operatorname{sgn} n \ \delta_{nn'}\} \otimes 1_R \otimes 1_T \end{aligned}$$

where

$$\boldsymbol{G} = \{g = \{g_{nm}^{\alpha\beta,ij}\} | g^\dagger = g, g^* = MgM^{-1}\}$$

and

$$\boldsymbol{H} = \{h \in \boldsymbol{G} | [h, \Lambda] = 0\}.$$

We next expand the action around its spatially constant saddle points. To this end we substitute a slowly fluctuating field configuration $Q(x) \equiv Q_0 + \delta Q(x)$ into the action,

and expand to lowest non-vanishing order in both δQ and ω. Taking into account that $Q(x)^2 = Q_0^2 = 1 \Rightarrow [\delta Q, Q_0]_+ = \mathcal{O}(\delta Q^2)$, we obtain

$$\begin{aligned}\delta S_1[Q,\hat{\mu}] = {} & \frac{\pi\nu}{8\tau}\,\mathrm{tr}\sum_q \delta Q_\phi(q)\,\delta Q_\phi(-q) \\ & - \frac{1}{16\tau^2\mathcal{V}}\,\mathrm{tr}\sum_q \delta Q_\phi(q)\,\delta Q_\phi(-q)\sum_k G^+(k)G^-(k+q) \\ & + \frac{\mathrm{i}}{2\mathcal{V}}\sum_k(\mathrm{Im}\,G^-(k))\,\mathrm{tr}\sum_x[+\mu-\hat{\mu}+\mathrm{i}\hat{\omega}]Q(x)+\cdots \end{aligned} \tag{16}$$

where

$$G^\pm = (\mu - H_K\big|_{\phi=0} \pm \mathrm{i}/(2\tau))^{-1}$$

$$Q_\phi(x) = \exp(\mathrm{i}\varphi(x)\sigma_3^{\mathrm{T}})Q\exp(-\mathrm{i}\varphi(x)\sigma_3^{\mathrm{T}})$$

and where $\varphi(x) = (\phi/\phi_0)2\pi x/L_\parallel$ is a 'gauge-transformed' Q-matrix, and $\mathcal{V} = L_\perp^{d-1}L_\parallel$ the system volume. In deriving (16) use has been made of the fact that owing to the invariance of the $\omega = 0$ action under the transformations $Q \to T_0^{-1}Q_0T_0$, $T_0 =$ constant, Q_0 may be set to Λ. We next have to perform the k-momentum summations, and it is this step of the derivation in which the dimensionality of the model plays a role. The actual calculation of the sums is a bit lengthy and has been deferred to the appendix. As a result we obtain

$$\frac{1}{\mathcal{V}}\sum_k \mathrm{Im}\,G^-(k) \equiv \pi\nu = \frac{1}{2ta^d} \tag{17}$$

$$\frac{1}{\mathcal{V}}\sum_k G^-(k)G^+(k+q) = 2\pi\nu\tau\left[1 + D\tau\left(q_\parallel^2 + \frac{1}{2d}q_\perp^2\right)\right] \tag{18}$$

where $D = 4t^2a^2\tau$ is the diffusion constant of the model. The momenta appearing in this expression are quantized in units of $\pi/L_\parallel$ and $\pi/L_\perp$ respectively. Assuming that $L_\parallel \gg (2d)^{1/2}L_\perp$, we may restrict ourselves to the consideration of the zero mode in the transverse direction, $q_\perp = 0$. Under this condition, the insertion of (18) into (16) yields

$$\delta S_1[Q,\hat{\mu}] = \frac{\pi S\nu}{2}\int_0^{L_\parallel}\mathrm{tr}\left(\frac{D}{4}\,\partial_\phi Q\,\partial_\phi Q + \mathrm{i}\left[\mu-\hat{\mu}+\mathrm{i}\hat{\omega}\right]Q\right)$$

where the gauge transformation has been reversed, a continuum limit in the longitudinal direction has been taken,

$$\partial_\phi = \mathrm{i}\,\partial + \frac{\phi}{\phi_0}\frac{2\pi}{L_\parallel}[\sigma_3^{\mathrm{T}},\ ]$$

is a 'covariant derivative', $S = N_\perp a^{d-1}$ denotes the transverse cross section of the ring, and the Q-fields depend only on the longitudinal coordinate. δS_1 is the action of a standard σ-model for a quasi-one-dimensional wire (cf. e.g. [37, 38] for comprehensive reviews). The dimensionality enters only through the definition of the cross section S.

At this point we might stop since the level number fluctuations of this model and their flux dependence are known [6, 34]. Nonetheless we shall carry the computation of $\overline{\delta N^2}$ on to its end in order to introduce the type of perturbation theory that will be applied to analyse the interacting model below.

After differentiation with respect to $\hat{\mu}$, the expectation value for the level number fluctuations takes the form

$$\overline{\delta N^2} = -\left(\frac{\pi\nu S}{2\beta}\right)^2 \lim_{N\to 0}\left\langle\int_0^{L_\parallel}\mathrm{tr}(QE_R^{11})\int_0^{L_\parallel}\mathrm{tr}(QE_R^{22})+\cdots\right\rangle \tag{19}$$

where $(E_R^{\beta\beta'})_{\alpha\alpha'} = \delta_{\beta\alpha}\delta_{\beta'\alpha'}, \alpha\alpha' = 1, \ldots, N$, the dots represent terms of the structure $\sim \mathrm{tr}(QE_R^{11}QE_R^{22})$ which do not give a significant contribution, and

$$\langle \cdots \rangle = \int \mathcal{D}Q \ \exp(-S_1[Q, \hat{\mu} = \mu \cdot 1_N])(\cdots).$$

To compute the functional average, we introduce the parametrization

$$Q = \begin{pmatrix} 1 & B \\ -B^\dagger & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \begin{pmatrix} 1 & B \\ -B^\dagger & 1 \end{pmatrix} \tag{20}$$

where the matrix structure corresponds to the sign of the Matsubara index, i.e. the matrix $B = \{B_{nm}\}$ has a positive (negative) index n (m). We next plug (20) into the functional and represent the action in terms of the field B. Schematically, we obtain

$$S_1[Q] = S_1^{(2)}[B] + S_1^{(4)}[B] + \cdots$$

where $S_1^{(2n)}[B]$ denotes the contribution of $2n$th order in B. To compute $\overline{\delta N^2}$ to leading order in $1/N_\perp$ we just need the quadratic term $S_1^{(2)}$ whose q-representation reads

$$\begin{aligned} S_1^{(2)}[B] &= \mathcal{V}\pi\nu \sum_{n>0,m<0} \sum_q \sum_{ij} \mathrm{tr}\left[B_{nm}^{ij}(q)\left[K_{nm}^{ij}(q)\right]^{-1}(B^\dagger)_{mn}^{ji}(q)\right] \\ K_{nm}^{ij}(q) &= \left[D\left(q + \frac{2\phi}{\phi_0}\frac{2\pi}{L_\perp}(i-j)\right)^2 + \omega_n - \omega_m\right]^{-1}. \end{aligned} \tag{21}$$

Combining equations (19) and (20) we obtain

$$\overline{\delta N^2} = -(T\pi\nu\mathcal{V})^2 \lim_{N\to 0} \left\langle \left[\sum_{n>0,m<0} \mathrm{tr}(B_{nm}^{12}(B^\dagger)_{mn}^{21} - (12 \leftrightarrow 21))\right]^2 \right\rangle_0 \tag{22}$$

where the superscripts 12 and 21 refer to the replica space and

$$\langle \cdots \rangle_0 = \int \mathcal{D}(B, B^\dagger) \ \exp(-S_1^{(2)}[B])(\cdots).$$

To compute (22) we employ Wick's theorem which states that the functional average is given by the sum of all total contractions, where individual contractions are defined by the rule

$$\left\langle \cdots \underbrace{B_{nm}^{\alpha\beta,ij}(q) \cdots \qquad \cdots (B^\dagger)_{m'n'}^{\beta'\alpha',j'i'}(q')} \cdots \right\rangle = \delta_{\alpha\alpha'}\delta_{\beta\beta'}\delta_{ii'}\delta_{jj'}\delta_{qq'}\delta_{nn'}\delta_{mm'}K_{nm}^{ij}(q)\langle \cdots \rangle \tag{23}$$

and an analogous equation for contractions between B and B ($B^\dagger$ and $B^\dagger$) which follows from $B_{nm}^{\alpha\beta,ij}(q) = M^{jj'}(B^\dagger)_{mn}^{\beta\alpha,i'j'}(M^{-1})^{i'i}$. The application of Wick's theorem to (22) gives the result

$$\overline{\delta N^2} = 2T^2 \sum_{n>0,m<0} \sum_q \left(D\left[q + \frac{2\phi}{\phi_0}\frac{2\pi}{L_\perp}\right]^2 + \omega_n - \omega_m\right)^{-2} + \cdots \tag{24}$$

where the dots denote flux-independent contributions. After analytic continuation from Matsubara to real frequencies we obtain equation (9) of reference [3]. In other words, the persistent current of the non-interacting disordered high-dimensional ring coincides with the current flowing in an analogous low-dimensional geometry. Taking this in combination with the findings of section 2, we may thus conclude that the dimensionality of the model does not play a significant role as long as the interaction is switched off.

4. Interaction and disorder

In the preceding section we have seen that the physics of the two non-interacting models, the low- and the high-dimensional one, is basically the same. We next enquire into the role of the Coulomb interaction in the high-dimensional model hoping that our findings will allow us to draw at least some qualitative conclusions as to the situation in the low-dimensional case.

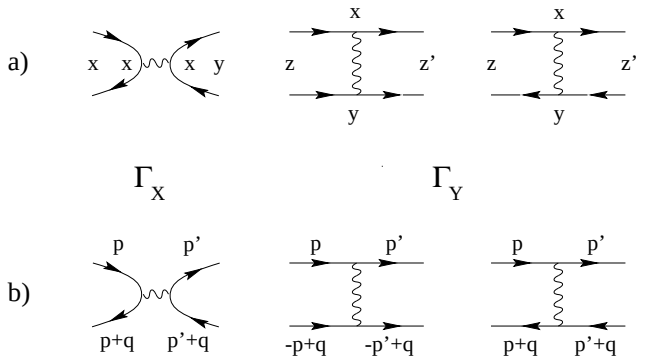

Figure 2. Effective Coulomb vertices in real space (a), and the momentum-space representation (b). The solid (wavy) lines represent electronic Green functions (the Coulomb amplitude U). The momentum q appearing in the bottom part of the figure is small in comparison with the momenta p and p' (cf. the corresponding remarks in the text).

The field theory that we are constructing is meant to describe the long-range phase-coherent propagation of electrons. In this context the (momentum representation of the) vertex representing the nearest-neighbour Coulomb interaction

$$S_2[\Psi] = \frac{\beta}{4}\sum_{p_1,p_2,p}\sum_{n_1,n_2,n}\sum_{\alpha}\Psi^{\alpha\dagger}_{n_1}(p_1)\Psi^{\alpha}_{n_2+n}(p_1+p)U(p)\Psi^{\alpha\dagger}_{n_2}(p_2)\Psi^{\alpha}_{n_2-n}(p_2-p)$$
$$U(p) = \frac{U}{d}\sum_{j=1}^{d}\cos(p_j a) \tag{25}$$

contains a lot of irrelevant information. Only those contributions where one of the frequency/momentum (p,ω) degrees of freedom appearing in the summation is small in the sense that $(p,\omega) < \mathcal{O}(l^{-1},\tau^{-1})$ can give rise to potentially relevant effects (l is the elastic mean free path of the model). To include these terms in our field theory, we proceed along the lines of references [30, 31]. In this paper we restrict ourselves to a brief review of the construction, and refer readers who are interested in details to the original reference. The interaction vertex is first approximated by

$$S_2[\Psi] = \frac{\beta}{4}\sum_{p,p',q}\sum_{n,n',m}\sum_{\alpha}\sum_{ij}\Psi^{\alpha,i}_{n}(p)\Psi^{*\alpha,i}_{n+m}(p+q)U(q)\Psi^{\alpha,j}_{n'+m}(p'+q)\Psi^{*\alpha,j}_{n'}(p')$$
$$-\,2\Psi^{\alpha,i}_{n}(p)\Psi^{*\alpha,j}_{n+m}(p+q)U(p'-p)\Psi^{\alpha,j}_{n'+m}(p'+q)\Psi^{*\alpha,i}_{n'}(p') \tag{26}$$

where the first (second) term represents the first term (second and third terms) shown in figure 2 [35] and $q < l^{-1}$. In other words, the vertex (26) is equivalent to (25) subject to the constraint that one of the momenta be small. To decouple these two terms by means of

a Hubbard–Stratonovich transformation, we introduce two auxiliary fields $X = \{X_m^{\alpha,i}(q)\}$ and $Y = \{Y_m^{\alpha,ij}(p,q)\}$ and define the functional averages

$$\begin{aligned}\langle\cdots\rangle_X &= \int \mathcal{D}X \;\; \exp(-\beta[X_m^{\alpha,i}(q)U(q)X_{-m}^{\alpha,j}(q)])(\cdots)\\ \langle\cdots\rangle_Y &= \int \mathcal{D}Y \;\; \exp(-\beta[Y_m^{\alpha,ij}(p,q)U(p-p')Y_{-m}^{\alpha,ji}(p',-q)])(\cdots)\end{aligned} \tag{27}$$

where a summation over all indices is understood. In analogy to the (disorder) decoupling scheme outlined in the preceding section, we eliminate the quartic terms (26) by means of a Gaussian shift, and integrate over the Ψ field. As a result we obtain

$$\overline{Z[\hat{\mu}]} = \int \mathcal{D}Q \;\; \exp\left(-\frac{\pi\nu}{8\tau}\,\mathrm{tr}\,Q^2\right)\left\langle \exp\left(\frac{1}{2}\,\mathrm{tr}\ln\left[G_0^{-1} + \frac{\mathrm{i}Q}{2\tau} + \sqrt{2}\mathrm{i}UX + \sqrt{8}UY\right]\right)\right\rangle_{X,Y}.$$

We next expand the logarithm to leading order in X and Y. This approximation is justified because these fields fluctuate on scales T (cf. equation (27)), whereas

$$\left(G_0^{-1} + \frac{\mathrm{i}}{2\tau}Q\right)^{-1} \sim 1/\tau \gg T.$$

The (Gaussian) integral over X and Y is then easily carried out, and we arrive at the effective action

$$\begin{aligned}S[Q] &:= S[Q, \hat{\mu} = \mu\cdot 1_N] = S_1[Q] + S_2[Q]\\ S_1[Q] &= \frac{\pi S\nu}{2}\int_0^{L_\parallel}\mathrm{tr}\left(\frac{D}{4}\,\partial_\phi Q\,\partial_\phi Q - \hat{\omega}Q\right)\\ S_2[Q] &= -\frac{S}{16\beta\tau^2}\sum_\alpha\sum_{ij}\sum_{nn'm}\int_0^{L_\parallel}\left[\Gamma_X Q^{\alpha\alpha,ii}_{n+m\,n}Q^{\alpha\alpha,jj}_{n'+m\,n'} - 2\Gamma_Y Q^{\alpha\alpha,ij}_{n+m\,n}Q^{\alpha\alpha,ji}_{n'-m\,n'}\right]\end{aligned}$$

where the coupling constants are given by

$$\begin{aligned}\Gamma_X &= \frac{1}{\mathcal{V}}\sum_{pp'}G^+(p)G^-(p)U(0)G^+(p')G^-(p')\\ \Gamma_Y &= \frac{1}{\mathcal{V}}\sum_{pp'}G^+(p)G^-(p)U(p-p')G^+(p')G^-(p').\end{aligned} \tag{28}$$

The calculation of the momentum sums in (28) is detailed in the appendix, and yields

$$\begin{aligned}\Gamma_X &= (2\pi\nu\tau)^2\mathcal{V}U\\ |\Gamma_Y| &< \frac{\pi^2}{d}(2\pi\nu\tau)^2\mathcal{V}U.\end{aligned} \tag{29}$$

At this point the dimensionality of the model begins to play a role. Equations (29) imply that the Coulomb vertices Γ_Y are negligibly small in the $d\to\infty$ limit. Whereas the precise computation of the coupling constants is somewhat tedious, the physical origin of the suppression factor $1/d$ appearing in (29) is easy to understand. We will come back to this point in the next section. Here we focus on the consequences of (29) for the perturbative evaluation of the model.

In analogy to previous analyses [7, 8], we compute the persistent current of the interacting model directly from the grand-canonical potential [36]. The expectation value that we have to compute thus reads

$$\overline{I(\phi)} = -\partial_\phi\overline{F(\phi)} = \beta^{-1}\lim_{N\to 0}N^{-1}\,\partial_\phi\int\mathcal{D}Q\;\mathrm{e}^{-S[Q]}.$$

We next subject this functional to the same perturbative expansion scheme as has been used to analyse the non-interacting field theory in the preceding section. To this end we insert the representation (20) into $S_2[Q]$, thereby generating a series

$$S_2[Q] = \sum_{n=1}^{\infty} S_2^{(2n)}[B].$$

In principle we should now expand the functional around the Gaussian term

$$\exp(-S_1^{(2)}[B] - S_2^{(2)}[B])$$

and apply Wick's theorem. Yet the complex index structure of

$$S_2^{(2)}[B] = -\frac{ST\pi^2\nu^2U\mathcal{V}}{4} \sum_{\alpha,ij} \sum_{\text{Mat.}} \int_0^{L_\parallel} B^{*\alpha\alpha,ii}_{n+m\,n}\, B^{\alpha'\alpha',i'i'}_{n'-m\,n'} \tag{30}$$

where

$$\sum_{\text{Mat.}} := \sum_{n,n'-m>0,\, n+m,n'<0}$$

suggests treating this term on the same footing as the higher-order terms $S^{(n>2)}[B]$ at the expense of a perturbation series which is to be continued to infinite order in $S_2^{(2)}[B]$. The fact that all of the $S_2^{(2)}[B]$ contribute to the same order in the small expansion parameter of the theory, the inverse of the channel number $1/N_\perp$, is easy to understand just from power counting: after contraction, each vertex power $(S^{(2n)})^l$ contributes a factor $\sim N_\perp^{l(1-n)}$. For $n = 1$ the l-dependence drops out, i.e. all powers $(S_2^{(2)})^l$ contribute to the same order of perturbation theory. The same argument tells us that vertices $S_2^{(2n>2)}$ give rise to small contributions of higher order in $1/N_\perp$.

Focusing on the $S_2^{(2)}$-contribution to the free energy, we obtain

$$\overline{F(\phi)} \simeq -\beta^{-1} \lim_{N\to 0} N^{-1} \left\langle \sum_{l=0}^{\infty} \frac{1}{l!} (S_2^{(2)}[B])^l \right\rangle_0 . \tag{31}$$

It is a straightforward matter to compute the total contraction of this series by Wick's theorem. Yet there is no need to do so, for all B-matrix elements appearing in (31) are diagonal in the T indices i, j (cf. equation (30)), which means that their contraction yields nothing but field-independent elements K^{ii} (cf. equations (21) and (23)). In other words, the dominant interaction-induced contribution to the free energy is insensitive to the magnetic flux, and does not affect the persistent current.

To understand this result in physical terms, one may visualize the perturbation expansion diagrammatically. Discussions of the relation between standard diagrammatic perturbation theory and the perturbative evaluation of the non-linear σ-model can be found in various places in the literature (cf. e.g. references [37, 39]). Here we merely note that every contraction of two matrices B^{ij} yields a diffusion (cooperon) pole K^{ii} ($K^{i\neq j}$) for identical (non-identical) indices i and j. The Matsubara frequencies/momentum determining the pole are specified by the index structure of the B-matrices according to (23). In this way the contraction of the interaction vertices $S_2^{(2)}$ can be identified as (RPA-corrected) Fock-type contributions to the free energy (cf. figure 3). Due to the absence of cooperons (diagrams with a maximal crossing of impurity lines), these diagrams are field independent. To obtain field-dependent contributions to the free energy, one might proceed to computing the contraction of higher-order vertices $S_2^{(2n>2)}$. According to the above power-counting argument, however, the resulting diagrams will be of higher order in $1/N_\perp$ and thereby

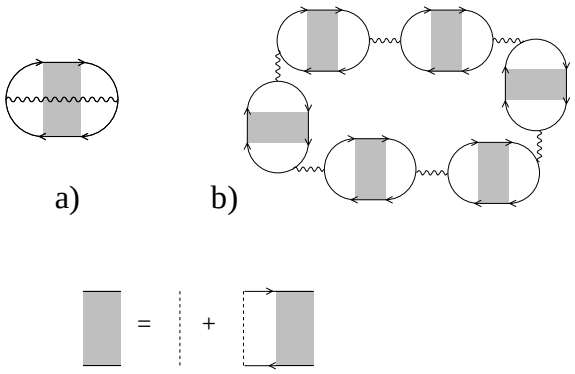

Figure 3. Fock-type contributions to the free energy arising from the contraction of one (a) or six (b) Coulomb vertices $S_2^{(2)}$ (cf. equation (31)). The shaded areas represent the diffusion poles K^{ii}. Their definition in terms of individual impurity scattering lines (dashed) is indicated in the bottom part of the figure.

negligibly small. We thus conclude that the Coulomb interaction in our model does not give rise to a significant contribution to the persistent current.

Why is this so? In the next section we address this question, and comment on the role of spin and dimensionality.

5. Qualitative considerations

In the non-interacting model, the persistent current depends sensitively on the global conservation of charge. Even minute charge fluctuations of order unity lead to its complete destruction. Under these circumstances, the Coulomb interaction—which counteracts charge fluctuations not only globally but even locally—should play a substantial role. Considerations of this type motivated the analysis of interaction effects in the present context [4].

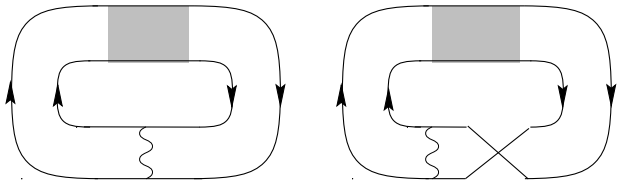

Figure 4. Hartree (left) and Fock (right) contributions to the free energy. Note that both diagrams involve the Coulomb vertex Γ_Y.

In a first approximation, local charge-neutrality-restoring processes are represented by Hartree- and Fock-type diagrams as shown in figure 4. In order to arrive at a physically complete picture, the simple Hartree–Fock scheme has to be supplemented by more complex processes [8]. Yet all these contributions to the free energy have in common that they involve the Coulomb vertex Γ_Y which is *absent* in the present model. In other words, the high-dimensional model does not account for the basic physical mechanism responsible for the suppression of charge fluctuations. It is thus no surprise that the Coulomb interaction does not play a role.

What is the physical reason for the smallness of the amplitude Γ_Y in the high-dimensional spinless model? Diagrams contributing to mesoscopic observables contain the Coulomb interaction in either of the forms shown in figure 2. The building blocks displayed in figure 2(a) are just the real-space representation of the Coulomb amplitudes Γ_X and Γ_Y. In high dimensions the single-particle Green functions appearing in these vertices decay rapidly in space ($G^{\pm}(x, y) \sim d^{-|x-y|/(2a)}$). Bearing in mind that the Coulomb interaction is spatially non-diagonal, one may see just by inspection that for any site configuration (x, y, z, z') two of the Green functions in the Γ_Y vertex have to be taken between spatially more remote arguments than in the analogous Γ_X diagram. This is the origin of the d^{-1}-suppression factor in (29). In other words, it is the spatial structure of the Coulomb interaction which is responsible for the smallness of Γ_Y. The nearest-neighbour modelling of the Coulomb interaction is in turn enforced by the absence of spin in our model. We are thus led to the conclusion that the inclusion the electron spin must be of crucial importance in the analysis of Coulomb interactions in the high-dimensional case.

In the above argumentation the dimensionality of the model played a central role. One might consider the subsequent discussion of the consequences for the persistent current as largely academic if it were not for the fact that the basic reason for the irrelevance of interactions (the suppression of Γ_Y in the absence of spin) generalizes to the more relevant cases of low-dimensional models close to half-filling. This may be understood by considering the momentum-space representation of the Coulomb amplitudes (cf. figure 2(b)) and its quantitative formulation (28). Making use of the definition of the Coulomb amplitude U in (25) and of the fact that the Green functions are even in the momentum arguments, the coupling constants can be rewritten as

$$\Gamma_{X/Y} \sim \left| \sum_p F(p) P_{X/Y}(p) \right|^2 \tag{32}$$

where $F(p) = |G^+(p)|^2$ is a function which is sharply peaked at the Fermi surface, $P_Y(p) = \exp(\mathrm{i}p_1 a)$, p_1 is the first (or any other) momentum component, and $P_X = 1$. Consider now a low-dimensional—say two-dimensional—model close to half-filling. In this case the Fermi surface takes the form of a square with corners (p_1, p_2) at $(\pi/a, 0), (0, \pi/a), (-\pi/a, 0)$, and $(0, -\pi/a)$. The average of the rapidly fluctuating phase factor P_Y over this 'Fermi square' is zero. We thus find that close to the half-filled case, Γ_Y is negligibly small in comparison with Γ_X. Note (i) that in the case of a Hubbard (site-diagonal) interaction the argument does not apply since $P_X = P_Y = 1$, and (ii) that the suppression mechanism is the less effective the more the filling deviates from 1/2. For example, for a quarter-filled band there are no significant differences between Γ_X and Γ_Y.

This simple argument indicates that the seemingly marginal difference between nearest-neighbour and on-site interactions can indeed be crucial for the analysis of interaction effects in weakly disordered metals.

6. Summary

In this paper we have introduced a model for the persistent current carried by spinless fermions moving in a ring with a high-dimensional cross section. The effects of both disorder and interactions were considered. It turned out that the non-interacting model shares practically all of its physical properties with the corresponding low-dimensional model systems; the effect of the dimensionality merely amounts to an increase in the number of transverse channels. To deal with the full problem, we adapted the Finkelstein theory of

interacting disordered Fermi systems to the case of spinless fermions in high dimensions. An application of this field theory to the calculation of the current showed that the latter is not affected by interactions in the limit $d \to \infty$. This result is most probably a consequence of the neglect of the electron spin.

We believe that the contribution of this paper is twofold. First, we have identified a physical mechanism which may be responsible for the importance of the role of the spin in the persistent-current problem. In both low and high dimensions the electron spin has to be incorporated in order to allow for the presence of site-diagonal contributions to the interaction Hamiltonian. These operators are essential for the formation of charge-neutrality-preserving contributions to the free energy which enhance the persistent current in real systems. Note that we cannot exclude the possibility of the existence of complementary mechanisms that are responsible for the relevance of the spin in low dimensions.

Second, the field theoretical formalism presented in this paper may serve as an efficient tool for the analysis of weakly disordered high-dimensional Fermi systems in general. (At any rate, the experience gained in 'low-dimensional' mesoscopic physics has shown that complex perturbative analyses are more conveniently performed by field theoretical methods rather than by direct diagrammatic calculations.) As for the specific problem of persistent currents, the next steps would be those of accounting for the electron spin and d^{-1}-corrections.

Acknowledgments

We thank G Bouzerar, G Uhrig, D Vollhardt and M Zirnbauer for instructive discussions, and M Backhaus for carefully reading the manuscript.

Appendix. Momentum summations

In the text we repeatedly encountered momentum summations of the type

$$\sum_{k_\perp} f(\epsilon_\perp(k_\perp))$$

where f is smooth. Due to the complex structure of the tight-binding Fermi surface, expressions of this kind are typically difficult to handle. In the case of *large* dimensionality, however, statistical arguments can be applied to simplify the computation of the sum. In this limit, the expression

$$\epsilon_\perp(k_\perp) = -\frac{\sqrt{2}t}{d}\sum_{i=1}^{d-1}\cos(k_i a)$$

can be interpreted as a sum of $d-1 \gg 1$ effectively random variables $\sim\cos(k_i d)$ [23]. According to the central-limit theorem, $\epsilon_\perp$ is a Gaussian-distributed variable centred around 0 with width $\sim t$. Replacing the momentum summation by an integral over the smooth distribution function of $\epsilon_\perp$, we obtain (7). Typically, the functions that we are going to encounter are sharply peaked around some value of $\epsilon_\perp$ well inside the band. Under these circumstances the Gaussian probability distribution in (7) can be replaced by a (technically easier to handle) box distribution of width $2t$. We next apply these ideas to the calculation of various momentum sums that appeared in the text.

A1. Calculation of $Im \sum_k G^-(k)$

The impurity-averaged Green functions entering this expression are defined as

$$G^{\pm}(k) = \left[\mu \pm \mathrm{i}/(2\tau) - \epsilon_{\parallel}(k_{\parallel}) - \epsilon_{\perp}(k_{\perp})\right]^{-1}.$$

Making use of the identity

$$\mathrm{Im}\, G^-(k) = G^+(k)G^-(k)/(2\tau)$$

we obtain

$$\mathrm{Im} \sum_k G^-(k) \simeq \frac{1}{2\tau}\frac{N_\perp}{2t}\sum_{k_\parallel,o}\int_{-t}^{t} \mathrm{d}\epsilon\ \frac{1}{\left|\mu(k_\parallel) + \mathrm{i}/(2\tau) - \epsilon\right|^2} \simeq \frac{\pi N_\perp}{t}\sum_{k_\parallel,o} 1 = \frac{N\pi}{2t} \tag{A1}$$

where N is the number of tight-binding states with energy smaller than μ, $\mu(k_\parallel) = \mu - \epsilon(k_\parallel)$, and $\sum_{k_\parallel,o} = \sum_{k_\parallel,|\mu(k_\parallel)|<t}$ is a sum over the open transverse channels. In the second equality we have made use of the fact that (except for those few values of $k_\parallel$ whose energy is close to the band edges, $t - |\mu(k_\parallel)| \simeq 1/\tau$) the ϵ-integration can be replaced by an unconstrained one. Equation (A1) leads to the first identity in (18).

A2. Calculation of $\sum_k G^-(k)G^+(k+q)$

We first expand this expression to lowest non-vanishing order in the small momentum $q \sim 1/l \ll k \sim 1/a$. As the result we obtain

$$\sum_k G^-(k)G^+(k+q) = \sum_k \Bigg(G^-(k)G^+(k) + 8t^2a^2\Bigg[\sin^2(k_d a)q_\parallel^2 + \frac{1}{2d}\sum_{i=1}^{d-1}\sin^2(k_i a)q_i^2\Bigg]G^-(k)G^+(k)^3\Bigg) + \cdots$$

where the ellipsis denotes terms which either vanish by symmetry (terms that are linear in q or proportional to $q_i q_j, i \neq j$) or oscillate rapidly as a function of the momentum k (terms containing factors $\cos(k_i a)$). We next observe that the $\sin^2(k_i a)$ factors average to $1/2$. The momentum summation over the Green functions can then be performed in complete analogy to the calculation in the preceding subsection. As a result we obtain the second line in (18).

A3. Calculation of Coulomb amplitudes

The amplitude Γ_X is readily calculated, as it decays into a product of factors which have already been computed in section 6:

$$\Gamma_X = \frac{U(0)}{\mathcal{V}}\left[\sum_k G^+(k)G^-(k)\right]^2 = U(2\pi\nu a^d\tau)^2.$$

The calculation of Γ_Y is more intricate. We first rewrite Γ_Y as

$$\Gamma_Y = \frac{U}{\mathcal{V}}\sum_{kk'}|G^+(k)|^2\cos((k-k')_1 a)|G^-(k')|^2 = \frac{U}{\mathcal{V}}\left(\sum_k |G^+(k)|^2\cos(k_1 a)\right)^2.$$

In the first equality use has been made of the fact that all cosine terms appearing in the definition (25) of the Coulomb amplitude give the same contribution (at this point we are dealing with a 'local' quantity and it is not necessary to single out the d-component). The second equality holds because the functions $G^{\pm}(k)$ are even in k. To compute the

momentum sum to lowest order in $1/d$, we expand the product of Green function in powers of $\cos(k_1 a)$:

$$|G^+(k)|^2 = |G^+(k)|^2\big|_{\cos(k_1 a)=0} + \left(\frac{2}{d}\right)^{1/2} t\,\partial_\mu |G^+(k)|^2\big|_{\cos(k_1 a)=0} \cos(k_1 a) + \cdots .$$

This leads to

$$\sum_k |G^+(k)|^2 \cos(k_1 a) \simeq \frac{1}{\sqrt{2d}} t\,\partial_\mu \sum_k |G^+(k)|^2$$

where we have used the facts that the $\cos(k_1 a)^2$ factor averages to $1/2$ and that the condition that the μ-derivative be evaluated at $\cos(k_1 a) = 0$ is inessential in the large d-limit. Noting that

$$\sum_k |G^+(k)|^2 = 2\pi\tau\nu(\mu)\mathcal{V}$$

where $\nu(\mu)$ is the (in principle μ-dependent) density of states, we arrive at

$$\Gamma_Y \simeq \frac{\mathcal{V}}{2d}(2\pi\tau t\,\partial_\mu \nu(\mu))^2.$$

Besides the $1/d$ factor, Γ_Y differs from Γ_X in the presence of the derivative operator $t\,\partial_\mu$ acting on $\nu(\mu)$. In our model, the density of states is μ-independent (apart from as regards the values of μ which are closer than $1/\tau$ to the band edges; cf. the remarks made at the beginning of this appendix), which means that the μ-derivative leads to still further suppression. All in all, we conclude that Γ_Y is negligibly small as compared with Γ_X.

References

[1] Levy L P, Dolan G, Dunsmuir J and Bouchiat H 1990 *Phys. Rev. Lett.* **64** 2074
[2] Hund F 1938 *Ann. Phys., Lpz.* **32** 102
[3] Altshuler B L, Gefen Y and Imry Y 1991 *Phys. Rev. Lett.* **66** 88
[4] Schmid A 1991 *Phys. Rev. Lett.* **66** 80
[5] von Oppen F and Riedel E K 1991 *Phys. Rev. Lett.* **66** 88
[6] Altland A, Iida S, Müller-Groeling A and Weidenmüller H A 1992 *Europhys. Lett.* **20** 155
Altland A, Iida S, Müller-Groeling A and Weidenmüller H A 1992 *Ann. Phys.* **219** 148
[7] Ambegoakar V and Eckern U 1990 *Phys. Rev. Lett.* **65** 381
Ambegoakar V and Eckern U 1991 *Phys. Rev. Lett.* **67** 3192
[8] Eckern U 1991 *Z. Phys.* B **82** 393
[9] Tsiper E V, Pikus F G and Efros A L 1994 *Solid State Commun.* **92** 485
[10] Berkovits R and Avishai Y 1995 *Europhys. Lett.* **29** 475
[11] Bouzerar G and Poilblanc D 1995 *Phys. Rev.* B **52** 10 772
[12] Ramin M, Reulet B and Bouchiat H 1995 *Phys. Rev.* B **51** 5582
[13] Müller-Groeling A, Weidenmüller H A and Lewenkopf C H 1993 *Europhys. Lett.* **22** 193
[14] Bouzerar G, Poilblanc D and Montambaux G 1994 *Phys. Rev.* B **49** 8258
[15] Berkovitz R 1993 *Phys. Rev.* B **48** 14 381
[16] Kato H and Yoshioka D 1994 *Phys. Rev.* B **50** 4943
[17] Doty A and Fisher D S 1992 *Phys. Rev.* B **45** 2167
[18] Mori H 1994 *Phys. Rev.* B **51** 12 943
[19] Römer R and Punnoose A 1995 *Phys. Rev.* B **52** 14 809
[20] Giamarchi T and Shastry B S 1995 *Phys. Rev.* B **51** 10 915
[21] As an aside, we mention that a diagrammatic Fock calculation utilizing the unrenormalized Thomas–Fermi interaction [7] has led to results which *do* conform with the experiment. However, this is probably a fortuitous coincidence.
[22] Metzner W and Vollhardt D 1989 *Phys. Rev. Lett.* **62** 324
[23] Müller-Hartmann E 1989 *Z. Phys.* B **74** 507
[24] Vlaming R, Uhrig G S and Vollhardt D 1992 *J. Phys.: Condens. Matter* **4** 773 (erratum **4** 10 103)

[25] Uhrig G S and Vlaming R 1993 *J. Phys.: Condens. Matter* **5** 2561
[26] Cheung H F, Gefen Y, Riedel E K and Shih W H 1988 *Phys. Rev.* B **37** 6050
Cheung H F, Gefen Y, Riedel E K and Shih W H 1988 *IBM J. Res. Dev.* **32** 34
[27] The Byers and Yang theorem [28] states that the persistent current is a ϕ_0-periodic quantity.
[28] Byers N and Yang C N 1961 *Phys. Rev. Lett.* **7** 46
[29] Negele J and Orland H 1988 *Quantum Many Particle Systems* (Redwood City, CA: Addison-Wesley)
[30] Finkelstein A M 1990 *Electron Liquids in Disordered Conductors (Soviet Scientific Reviews 14)* ed I M Khalatnikov (New York: Harwood Academic)
[31] Finkelstein A 1983 *JETP Lett.* **37** 517
Finkelstein A 1984 *Sov. Phys.–JETP* **59** 212
[32] Oppermann R 1985 *Z. Phys.* B **61** 89
[33] Note that in the coordinate basis the time-reversal operation amounts to a complex conjugation.
[34] Altland A, Efetov K B and Iida S 1993 *J. Phys. A: Math. Gen.* **26** 3545
[35] Due to the symmetry law (12), the second term on the rhs of (26) represents both the first and the second term in figure 2(b).
[36] Coulomb interactions tend to suppress fluctuations in the charge density. For this reason, one may anticipate that it is not necessary to implement the condition of global charge conservation manually as it was in the case of the non-interacting model. Previous calculations which gave a non-vanishing result in the interacting/grand-canonical case support this supposition.
[37] Efetov K B 1983 *Adv. Phys* **32** 53
[38] Efetov K B 1997 *Supersymmetry in Disorder and Chaos* (New York: Cambridge University Press)
[39] Altshuler B L, Kravtsov V E and Lerner I V 1991 *Mesoscopic Phenomena in Solids* ed B L Altshuler, P A Lee and R A Webb (Amsterdam: Elsevier)